Telecommunications and Radio Engineering, 72 (18):1651-1663 (2013)

CLIFFORD ALGEBRA IN MULTIPLE-ACCESS NOISE-SIGNAL COMMUNICATION SYSTEMS

M.Ye. Ilchenko[1], T.N. Narytnik[2♦], & R.M. Didkovsky[3]

[1]Research Institute for Telecommunications, NTUU "KPI"
37, Peremogy ave., 03056 Kyiv-056, Ukraine

[2]Institute of Electronics and Communications at UASNP
2-B, 50 Years of October Ave., 03148 Kyiv, Ukraine

[3]Cherkass State University of Technology
460, Shevchenko boulevard, 18006 Cherkassy, Ukraine

♦Address all correspondence to T.N. Narytnik E-mail: iec@naverex.kiev.ua

The present paper discloses that the system of matrices for orthogonal skew-symmetric anticommuting operators is apt to generate a Clifford algebra representation in terms of matrices. The statistical properties of this particular system of operators as applied to a random vector have been examined. The operator system obtained is feasible for producing a set of orthogonal signals in the multiple-access noise-signal communication system.

KEY WORDS: *the Clifford algebra, tensor product of matrices, random vector, correlation factor, noise-signal phase-shift keying communication system, code division of channels*

1. INTRODUCTION

The state-of-the art communication systems incorporates a number of critical features that have to be necessarily taken into account. The matter is that highly stringent requirements are now imposed upon these system not only as regards the authenticity of data and their transfer rate but also in terms of protecting some transmitted information against an unauthorized access, let alone the environmental safety [1-3].

The authors in [3] came up with the following concept. Their suggestion was that in a communication system a chaotic signal should be used as an informative carrier. In contrast to conventional communication systems where a reference signal is produced on a receiving side, the transmitter of this particular system is set to radiate a sequence of pairs of chaotic signal fragments. Each pair matches a single information bit.

The first fragment in the pair is a reference signal, the second one is an information signal given that an actual bit is equal to “1”, the information signal is a replica of the reference signal, and once it is equal to “0”, it is held to be opposite to the reference signal. The main feature of this system is the feasibility of incoherent signal reception.

Furthermore, in the above type of the system, a noise source can be made use of to generate signals instead of a dynamic chaos oscillator. By noise we imply that a certain broadband random process is realized.

Thus, by so doing one can build a communication system involving a noise carrier [4]. We will consider the signals to be generated and processed in the system at hand by digital methods in a discrete time. The reference signal will then take the form of vector $\overline{x} = (x_1, x_2, ..., x_n)$, whereas the transmission signal that persists on a signal bit interval is written as:

$$y_k = \begin{cases} x_k, & k = 1, 2, ..., n, \\ \alpha \cdot x_{k-n}, & k = n+1, n+2, ..., 2n, \end{cases}$$

where $\alpha = 1$, if the bit “1” is transmitted at the given instant of time, and $\alpha = -1$ if the current bit is equal to “0”. The dimensionality of vector n is determined by the number of clock time intervals Δt of the ADC-DAC system, these intervals fitting into a half of the bit interval of length $T/2$.

Thus, over the length of a single bit interval the transmitter signal can be given as a cortege of vectors

$$\overline{y} = \left(\overline{x}, \alpha\overline{x}\right).$$

Let us regard the random process that generate the reference signal $\overline{x}$ as the one meeting several additional condition: i) the given process is stationary; ii) any one-dimensional distribution of the process is symmetrical about zero; iii) the process correlation interval is far shorter than the clock time interval Δt.

Under these conditions the value of $\overline{x}$ can be thought of as a random vector the coordinates of which are the realizations of n of equally distributed independent random quantities. By virtue of the fact that these quantities are symmetrically distributed with respect to zero, their mathematical expectation is equal to zero.

As the signal passes through the communication channel, it is impacted by the additive noise. We will take advantage of realizing the steady broadband Gaussian process with zero mathematical expectation as a mathematical model. In the discrete time, on the bit interval T, we obtain two vectors that are defined as noise, i.e., $\overline{n}_1$ acting upon the reference signal and $\overline{n}_2$ affecting the information signal.

On a receiving side we calculate the scalar product

$$r = < \overline{x} + \overline{n}_1,\ \alpha\overline{x} + \overline{n}_2 >$$

whose sign specifies the decision about “0” or “1”.

2. PROBLEM STATEMENT

According to [5] a proposal was advanced to apply permutations of discrete elements of signal $\overline{y}$. This procedure affords two opportunities: i) to diminish the correlation between the reference and data signals in an effort to improve the data protection level against any external interferences; ii) to build a multiple-access communication system with a code division of channels.

A specific rearrangement of the vector-signal, which tends to substantially decrease an absolute quantity of the product $<\overline{x}, \alpha\overline{x}>$ is performed in a simplex regime on a receiving side.

Any outside man who is incompetent to go by the permutation rules is not only unable to gain an access to information, but rather he is totally incapable of determining the transfer rate of a signal and its structure. Neither spectral nor autocorrelation analysis will ever reveal any periodicity in a transformed signal.

The receiver accomplishes an inverse permutation of coordinates prior to calculating the value of r .

Each transmitter-receiver pair in the multiple-access system relies upon the permutation rule of its own. This permits of carrying out a code division of channels. At the same time the authors in [5] demonstrate that the increase in the number of communication channels concurrently in use will bring about a dramatic reduction in the systems noise immunity.

A possibility for constructing a different transport system capable of enchaining the noise immunity is shown in [6]. However, the issue pertinent to validating the efficiency of this system of transformation, investigating its properties as well as finding a maximal set of transformations remains open.

Thus, a primary goal of the present work is to elaborate the technique for development of a maximal system of linear transformation of a discrete signal. This system is to contribute toward generating a set of orthogonal signals to be utilized in the multiple-access communication system.

3. PROBLEM SOLVING

At the very outset, let us have the problem formalized mathematically. Assume R^n to be the n-dimensional Euclidean space, while $\overline{x}$ is the fixed null vector of this space, which is specified by its coordinates in a standard basis. It is necessary to construct a maximal system of linear transformations $A_1, A_2, ..., A_q$ for which the following conditions are met:

$$\left\langle A_i\overline{x}, \overline{x}\right\rangle = 0, \quad i = 1, 2, .., q , \tag{1}$$

$$\left\langle A_i\overline{x}, A_j\overline{x}\right\rangle = |\,\overline{x}\,|^2 \cdot \delta_{ij}, \; i = 1, 2, .., q, \; j = 1, 2, ..., q , \tag{2}$$

where δ_{ij} is the Kronecker symbol:

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

Since it is essential that the signal energy be conserved for constructing the orthogonal system of signals, the following equality has to be satisfied:

$$\left| A_i \overline{x} \right| = \left| \overline{x} \right|, \; i = 1, \ldots, q .$$

This means that a search for transformation A_i needs to be made in a group of orthogonal transformations $O(n)$.

In view of [7], it may be argued that A_i is for the orthogonal skew-symmetric (sign-changing) operators.

On the strength of operators orthogonality we have

$$A_i^{-1} = A_i^*, \; i = 1, 2, \ldots, q ,$$

And by virtue of skew symmetry

$$A_i^* = -A_i, \; i = 1, 2, \ldots, q ,$$

where A_i^* is the conjugation operator to A_i.

Whereupon we have

$$A_i^2 = A_i \cdot A_i = A_i \cdot \left(-A_i^*\right) = -A_i \cdot A_i^{-1} = -E, \; i = 1, 2, \ldots, q , \tag{3}$$

where E is the identical spatial transformation R^n. Thus, the square of each operator of the system is the antipodal mapping.

In view of condition (2), at $i \neq j$, we obtain

$$\left\langle A_i \overline{x}, A_j \overline{x} \right\rangle = \left\langle \overline{x}, A_i^* A_j \overline{x} \right\rangle = \left\langle \overline{x}, -A_i A_j \overline{x} \right\rangle = -\left\langle \overline{x}, A_i A_j \overline{x} \right\rangle = 0 ,$$

i.e., conditions (1) is met for the product $A_i A_j \; (i \neq j)$. Thus, any pairwise product of operators for the system is a skew-symmetric orthogonal operator as well.

Upon having done a series of simplifications in the equality

$$\begin{aligned} &\left(A_i A_j\right)^* = -A_i A_j, \; A_j^* A_i^* = -A_i A_j, \; \left(-A_j\right)\left(-A_i\right) = -A_i A_j, \\ &A_j A_i = -A_i A_j, \end{aligned} \tag{4}$$

we find that the pairwise product of the systems operators is anticommutative.

Now consider the unit vector $\overline{x}_0$ of the vector $\overline{x}$. If $\overline{x}$ traverses the space R^n (with the exception of the null vector), then $\overline{x}_0$ describes the sphere S^{n-1}. In this instance each mapping $A_i\overline{x}_0$, which satisfies (1), generates the field of vectors tangent to the sphere S^{n-1}. In [8] it is shown that the field being tangents to the sphere of vectors is available only when the dimensionality of the sphere $n-1$ is add. That signifies that the sought-for system of linear operators can be constructed in the even-dimensional space only.

Each operator of the system has a basis in which its matrix takes on the following form [7]

$$\begin{pmatrix} 0 & 1 & 0 & 0 & \ldots & 0 & 0 \\ -1 & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & -1 & 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & 0 & 0 & \ldots & -1 & 0 \end{pmatrix}.$$

In this basis an operator allows performing the transformations of coordinates using the formula:

$$(x_1, x_2, x_3, x_4, \ldots, x_{n-1}, x_n) \to (x_2, -x_1, x_4, -x_3, \ldots, x_n, -x_{n-1}).$$

In geometrical point of view this operator works as follows: the basis is split into the pairs of vectors and in each two-dimensional plane, which is specified by an appropriate pair, the turn is made by $\pi/2$.

Now consider the linear combination of the operators

$$\alpha_1 A_1 + \alpha_2 A_2 + \ldots + \alpha_q A_q$$

with real coefficients α_1, α_2, $\ldots$, α_q. A set of all such linear combinations makes up the real Euclidean space.

Now we will find the quadratic form of this space, which is consistent with the algebraic operators:

$$\left(\alpha_1 A_1 + \alpha_2 A_2 + \ldots + \alpha_q A_q\right)^2 = \alpha_1^2 A_1^2 + \alpha_2^2 A_2^2 + \ldots + \alpha_q^2 A_q^2 +$$
$$+\alpha_1\alpha_2 A_1 A_2 + \alpha_1\alpha_3 A_1 A_3 + \ldots + \alpha_1\alpha_3 A_3 A_1 + \alpha_1\alpha_2 A_2 A_1.$$

Taking into account of equalities (3) and (4) we arrive at

$$\left(\alpha_1 A_1 + \alpha_2 A_2 + \ldots + \alpha_q A_q\right)^2 = -\alpha_1^2 E - \alpha_2^2 E - \ldots - \alpha_q^2 E +$$

$$+\alpha_1\alpha_2 A_1 A_2 + \alpha_1\alpha_3 A_1 A_3 + \ldots - \alpha_1\alpha_3 A_1 A_3 - \alpha_1\alpha_2 A_1 A_2 =$$

$$= -\left(\alpha_1^2 + \alpha_2^2 + \ldots + \alpha_q^2\right) E.$$

According to [9], the system of operators $A_1, A_2, \ldots, A_q$ can subsequently be thought of as the basic elements of Clifford algebra $Cl(0,q)$ with positive signature $p=0$ and negative signature q

The technique for constructing the real irreducible matrix representations of basis elements of the maximal Clifford algebra with a preset signature can be found [10]. It is to be noted that all the matrixes already constructed in this fashion are of $2^m \times 2^m$ dimension and, as a consequence, we first focus our attention upon dimensionality spaces of $n = 2^m$.

Here again we should stress the point that in order to solve the problem that has been formulated in our work, our immediate concern will only be those maximal Clifford algebras that, with a fixed n, have the greatest negative signature. As will be apparent from the analysis of the paper in [10], we are in a position to obtain an appropriate sequence of maximal algebras, the beginning of which is listed in Table 1

TABLE 1: Maximal Clifford algebras with the greatest negative signatures at a specified space dimension n

m	1	2	3	4	5	6	7	8	9	…
n	2	4	8	16	32	64	128	256	512	…
Sign.	(0.1)	(0.3)	(0.7)	(1.8)	(0.9)	(0.11)	(0.15)	(1.16)	(0.17)	…

As it is evident from Table 1, the negative signatures form a sequence all the members of which can be obtained by adding the number multiple of 8 for the first four elements of the given sequence in the order of 1, 3, 7, 8. It is precisely this sequence that defines the maximal number of q elements of the sought-for system of transformations.

Now we thick it fit to scrutinize the technique of constructing the matrix representations of basis Clifford algebra elements (this technique is suggested in [10]).

At the very outset we shall get down to the simplest examples. The Clifford algebra $Cl(0,1)$ is isomorphic into a set of complex numbers. It can be represented conventionally by means of an imaginary unit i, or else via a real matrix.

$$\boldsymbol{\tau}_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Henceforward, in our analysis, we shall have to fall back upon a real irreducible representation of the algebra $Cl(2,1)$:

$$\boldsymbol{\tau}_0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \boldsymbol{\tau}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\tau}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Using the introduced notation we shall find that the Clifford algebra $Cl(0,3)$, which is isomorphic to the body of quanternious may be realized by means of three 4 x 4 matrices:

$$\boldsymbol{\gamma}_{(0,3),1} = \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \; \boldsymbol{\gamma}_{(0,3),2} = \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{\gamma}_{(0,3),3} = \mathbf{E}_2 \otimes \boldsymbol{\tau}_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

In these formulas the symbol $\otimes$ demotes the tensor (Kronecker-type) product of matrices, whereas $\mathbf{E}_n$ (here and henceforth) is the n by n unit matrix.

The algebra $Cl(0,7)$ can be represented by means of seven 8 x 8 matrices. Notating these matrices in explicit form appears to be rather cumbersome, therefore we only present the formulas to set up these matrices:

$$\boldsymbol{\gamma}_{(0,7),1} = \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_1 \otimes \mathbf{E}_2, \; \boldsymbol{\gamma}_{(0,7),2} = \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_2 \otimes \mathbf{E}_2, \; \boldsymbol{\gamma}_{(0,7),3} = \mathbf{E}_2 \otimes \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_1,$$

$$\boldsymbol{\gamma}_{(0,7),4} = \mathbf{E}_2 \otimes \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_2, \; \boldsymbol{\gamma}_{(0,7),5} = \boldsymbol{\tau}_1 \otimes \mathbf{E}_2 \otimes \boldsymbol{\tau}_0, \; \boldsymbol{\gamma}_{(0,7),6} = \boldsymbol{\tau}_2 \otimes \mathbf{E}_2 \otimes \boldsymbol{\tau}_0,$$

$$\boldsymbol{\gamma}_{(0,7),7} = \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_0 \otimes \boldsymbol{\tau}_0.$$

If $\boldsymbol{\gamma}_{(p,q),i}$ $(i = 1, 2, \ldots, p+q)$ are the Clifford gamma-matrices of the n x n dimensionality $Cl(p,q)$, than the gamma-matrices of algebra $Cl(p+1, q+1)$ have $2n \times 2n$ dimensionality and can be obtained from the formulae

$$\begin{aligned} &\boldsymbol{\gamma}_{(p+1,q+1),\, i} = \boldsymbol{\tau}_1 \otimes \boldsymbol{\gamma}_{(p,q),i}, \; i = 1, 2, \ldots, p+q, \\ &\boldsymbol{\gamma}_{(p+1,q+1),\, p+q+1} = \boldsymbol{\tau}_0 \otimes \mathbf{E}_n, \; \boldsymbol{\gamma}_{(p+1,q+1),\, p+q+2} = \boldsymbol{\tau}_2 \otimes \mathbf{E}_n. \end{aligned} \tag{5}$$

If we use formula (5) to the gamma-matrices of $Cl(0,7)$ algebra then we get 16 x 16 matrices that realize $Cl(1,8)$. Here it is to be noted that all skew-symmetric matrices will be numbered as 1 to 8, whereas the matrix $\boldsymbol{\gamma}_{(1,8),9}$ will be symmetric (with a positive signature).

Now that the representations for the algebra $Cl(0,1), Cl(0,3)$ and $Cl(0,7)$ are made available to us, we can notate the general formulae for a series of the Clifford algebras $Cl(0,k+8n)$, where $k=1,3,7$. Let $\boldsymbol{\gamma}_{(0,k),i}$ be the realization of $Cl(0,k)$ for $k=1,3,7$ then we have

$$
\begin{aligned}
\boldsymbol{\gamma}_{(0,k+8n),i} &= \boldsymbol{\gamma}_{(0,k),i} \otimes \boldsymbol{\gamma}_{(1,8),9} \otimes \ldots \quad \ldots \ldots \otimes \boldsymbol{\gamma}_{(1,8),9}, \\
& \qquad i = 1,2,\ldots,k, \\
\boldsymbol{\gamma}_{(0,k+8n),k+j} &= \mathbf{E}_{k+1} \otimes \boldsymbol{\gamma}_{(1,8),j} \otimes \mathbf{E}_{16} \otimes \ldots \quad \ldots \ldots \otimes \mathbf{E}_{16}, \\
\boldsymbol{\gamma}_{(0,k+8n),k+8+j} &= \mathbf{E}_{k+1} \otimes \boldsymbol{\gamma}_{(1,8),9} \otimes \boldsymbol{\gamma}_{(1,8),j} \otimes \mathbf{E}_{16} \otimes \ldots \quad \ldots \ldots \otimes \mathbf{E}_{16}, \\
\boldsymbol{\gamma}_{(0,k+8n),k+16+j} &= \mathbf{E}_{k+1} \otimes \boldsymbol{\gamma}_{(1,8),9} \otimes \boldsymbol{\gamma}_{(1,8),9} \otimes \boldsymbol{\gamma}_{(1,8),j} \otimes \mathbf{E}_{16} \otimes \ldots \ldots \ldots \otimes \mathbf{E}_{16}, \\
\ldots \quad \ldots & \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
\boldsymbol{\gamma}_{(0,k+8n),k+8(n-1)+j} &= \mathbf{E}_{k+1} \otimes \underbrace{\boldsymbol{\gamma}_{(1,8),9} \otimes \ldots \quad \ldots \otimes \boldsymbol{\gamma}_{(1,8),9} \otimes \boldsymbol{\gamma}_{(1,8),j}}_{n}, \\
& \qquad j = 1,2,\ldots,8.
\end{aligned}
$$

It is just in this particular way that one is able to deduce the realizations of all algebras from Table 1, except for the series of $Cl(1,8n)$. The γ -matrices of the Clifford algebra for $Cl(1,8n)$ are obtained from formula (5) in terms of realizing the algebra $Cl(0,7+8(n-1))$. Thus, we have a chance to develop the explicit formulas for finding the real matrix representation of any Clifford algebra included in Table 1.

It is well to bear in mind that the notation of operators (which match the γ - matrices of Clifford algebra) using the coordinate transformation formulas is by far more compact as compared to the matrix-represented versions. This is accounted for by matrix sparseness of the given operators. Examples of this type of reprentation for algebras $Cl(0,1), Cl(0,3), Cl(0,7)$ and $Cl(1,8)$ are listed in Table 2. As a result, we have the systems of transforms for the space dimensions amounting to 2, 4, 8 and 16, respectively.

As seen from the above Table 2, the formulas of coordinate transformation of the operators for these particular systems are notated as sign-changing permutations. Representing the operators by way of sign-changing permutations is not only more compact as compared to the matrix representation (it requires less memory space to retain information), and also it is more efficient computationally.

Notice again that the systems of transformations (which we are seeking) will incorporate the negative-signature operators only (i.e., those for which condition (3) is met). To be more specific, the transformation numbered as 9 for 16-measurable space, even if it is a basic element of the Clifford algebra $Cl(1,8)$, is beyond the scope of further analysis. In terms of formalism, it is indicative of the change-over to the nonmaximal algebra $Cl(0,8)$ with a purely negative signature.

We shall credit the system of transformations obtained in tabulated form as the orthogonal sign-changing permutations.

Note that in the case where the space dimension is equal to $n \neq 2^m$, it is necessary to perform bundling of this particular space into subspaces whose dimensions correspond to the power of 2.

TABLE 2: Operators represented by coordinate transformation formulas

n	i	$A_{(p,q),i}$
2	1	$(x_2, -x_1)$
4	1	$(x_4, x_3, -x_2, -x_1)$
	2	$(x_3, -x_4, -x_1, x_2)$
	3	$(x_2, -x_1, x_4, -x_3)$
8	1	$(x_7, x_8, x_5, x_6, -x_3, -x_4, -x_1, -x_2)$
	2	$(x_5, x_6, -x_7, -x_8, -x_1, -x_2, x_3, x_4)$
	3	$(x_4, x_3, -x_2, -x_1, x_8, x_7, -x_6, -x_5)$
	4	$(x_3, -x_4, -x_1, x_2, x_7, -x_8, -x_5, x_6)$
	5	$(x_6, -x_5, x_8, -x_7, x_2, -x_1, x_4, -x_3)$
	6	$(x_2, -x_1, x_4, -x_3, -x_6, x_5, -x_8, x_7)$
	7	$(x_8, -x_7, -x_6, x_5, -x_4, x_3, x_2, -x_1)$
16	1	$(x_{15}, x_{16}, x_{13}, x_{14}, -x_{11}, -x_{12}, -x_9, -x_{10}, x_7, x_8, x_5, x_6, -x_3, -x_4, -x_1, -x_2)$
	2	$(x_{13}, x_{14}, -x_{15}, -x_{16}, -x_9, -x_{10}, x_{11}, x_{12}, x_5, x_6, -x_7, -x_8, -x_1, -x_2, x_3, x_4)$
	3	$(x_{12}, x_{11}, -x_{10}, -x_9, x_{16}, x_{15}, -x_{14}, -x_{13}, x_4, x_3, -x_2, -x_1, x_8, x_7, -x_6, -x_5)$
	4	$(x_{11}, -x_{12}, -x_9, x_{10}, x_{15}, -x_{16}, -x_{13}, x_{14}, x_3, -x_4, -x_1, x_2, x_7, -x_8, -x_5, x_6)$
	5	$(x_{14}, -x_{13}, x_{16}, -x_{15}, x_{10}, -x_9, x_{12}, -x_{11}, x_6, -x_5, x_8, -x_7, x_2, -x_1, x_4, -x_3)$
	6	$(x_{10}, -x_9, x_{12}, -x_{11}, -x_{14}, x_{13}, -x_{16}, x_{15}, x_2, -x_1, x_4, -x_3, -x_6, x_5, -x_8, x_7)$
	7	$(x_{16}, -x_{15}, -x_{14}, x_{13}, -x_{12}, x_{11}, x_{10}, -x_9, x_8, -x_7, -x_6, x_5, -x_4, x_3, x_2, -x_1)$
	8	$(x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, -x_1, -x_2, -x_3, -x_4, -x_5, -x_6, -x_7, -x_8)$
	(9)	$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, -x_9, -x_{10}, -x_{11}, -x_{12}, -x_{13}, -x_{14}, -x_{15}, -x_{16})$

Our next objective is to analyze the statistical properties of transformation systems obtained and to compare them with the properties of permutations (the transformations proposed in [5] for applications in communication systems).

Based upon the fact that both types of transformations are the orthogonal operators it may be argued that their application will not affect the vector norms.

It should be emphasized that to maintain the mean value and the dispersion of a vector-argument is likewise evident in terms of performing permutation operations.

However, in the orthogonal sign-changing permutation an $A_{(p,q),i}\overline{x}$ has a different mean value of coordinates as compared to an initial vector. Hence, the dispersion will undergo certain changes. Now let us look into the impact these changes is likely to have upon the probability distribution of the average and the dispersion.

Our further studies will be pursued through the use of a simulation computer model in the Mathcad system. Now generate a sample of a normal random value of volume n and apply the transformation randomly sampled from the system which complies with the given value of n. We shall calculate the mean value and the dispersion for the initial vector and its image. The experiment will be repeated 10^6 times. The histograms showing the mean value and dispersion samples deduced are illustrated in Fig. 1.

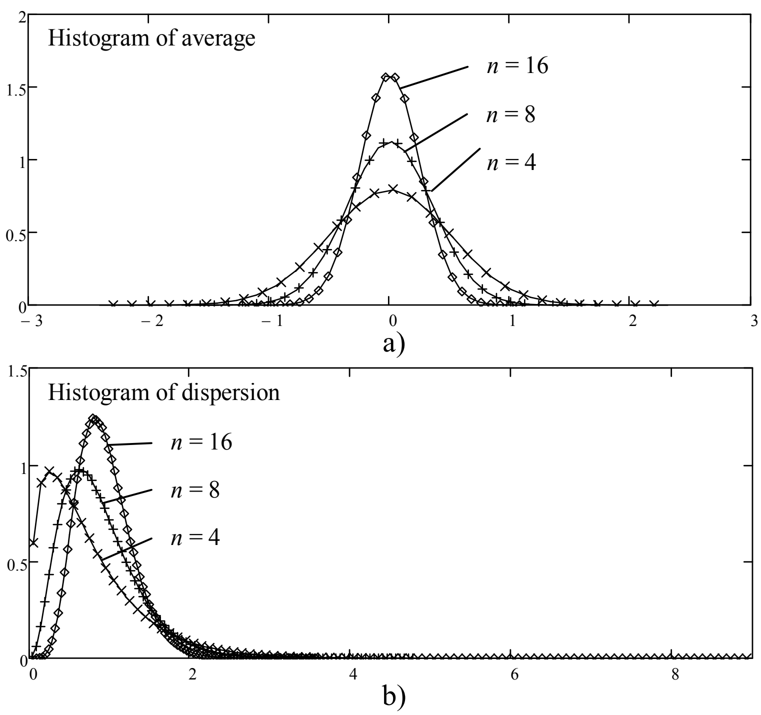

FIG. 1: The histograms of the mean value (a) and dispersion (b) of random vector $\overline{x}$ and its image in the orthogonal sign-changing permutations

Referring to the above Figure, the broken lines correspond to the histograms for the mean value and dispersion whereas the points are indicative of the histograms for the average and the dispersion of images of this vector $A_{(p,q),i}\overline{x}$. The computations were performed in relation to the space dimension of $n = 4,8,16$.

As will be apparent from the above graph, in terms of statistics, it seems impossible to distinguish the distributions of basic pointwise characteristics)of the average and the dispersion) for vector $\overline{x}$ and vector $A_{(p,q),i}\overline{x}$. That signifies that the influence of transforms $A_{(p,q),i}$ upon these characteristics of a signal is not a demarking factor in the data transfer. This fact hold good not only for the normal coordinate distribution of vector $\overline{x}$, but also for any distribution symmetrical with respect to zero.

If $\overline{x}$ is a discrete delta-correlated signal, then neither application of permutation nor that of sign-changing permutation will lead to the variations in the correlation properties inside that signal.

However, the most intriguing issue is the orthogonality between vector $\overline{x}$ and its images as well as pairwise between the different images of a single vector.

And now let us get down to analyzing this issue by means of statistical procedures. We shall compute two matrices far the random

$$K[i,j] = \cos\left(A_i\overline{x}, A_j\overline{x}\right),\ R[i,j] = \rho\left(A_i\overline{x}, A_j\overline{x}\right),\ i = 0,1,...,q,\ \ j = 0,1,...,q,$$

where $A_0\overline{x}$ is considered to mean the vector $\overline{x}$ itself, whereas $\rho\left(A_i\overline{x}, A_j\overline{x}\right)$ is the correlation factor between the vectors $A_i\overline{x}$ and $A_j\overline{x}$.

For comparison purposes, we shall decide on the permutations whose matrix representation can be obtained by taking a module of the corresponding element in the operator matrix $A_{(p,q),i}$ (see Table 2).

Below are given the examples of matrices K and R (K_1 and R_1 for orthogonal sign-changing permutations, K_2 and R_2 for permutations).

$$K_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \qquad R_1 = \begin{pmatrix} 1 & 0.342 & -0.018 & -0.06 \\ 0.342 & 1 & 0.265 & 0.863 \\ -0.018 & 0.265 & 1 & -0.047 \\ -0.06 & 0.863 & -0.047 & 1 \end{pmatrix};$$

$$K_2 = \begin{pmatrix} 1 & -0.925 & -0.69 & 0.709 \\ -0.925 & 1 & 0.709 & -0.69 \\ -0.69 & 0.709 & 1 & -0.925 \\ 0.709 & -0.69 & -0.925 & 1 \end{pmatrix}; \quad R_2 = \begin{pmatrix} 1 & -0.971 & -0.73 & 0.702 \\ -0.971 & 1 & 0.702 & -0.73 \\ -0.73 & 0.702 & 1 & -0.971 \\ 0.702 & -0.73 & -0.971 & 1 \end{pmatrix}.$$

Now choose some nondiagonal element in these matrices and proceed to perform the computations for this element by a factor of 10^6. Figure 2 gives the histograms for the samples already obtained. We shall perform the computations for the space dimensions of $n = 4,8,16$.

In keeping with the properties of orthogonal sign-changing permutations, all nondiagonal elements of matrix K_1 (angle cosines between images) are strictly equal to zero. Therefore the histogram for the given index is not shown in the above Figure.

Figure 2 indicates that, as the space dimension *n* grows, the distributions of the angle cosine and the correlation factor between the images, with different permutation being applied, also get concentrated close to zero. However, this process occurs at a far slower rate them for the correlation factor in the application of sign-changing permutations (in this case the angle cosine is always equal to zero).

The rigorous orthogonality of signals concurrently radiated in different channels of a single communication system tends to keep the level of inter-system interferences to a minimum. This is just a clear indication of the advantage offered by the noise immunity of the communication system that rely upon the systems of orthogonal sign-changing permutations for code division of channels.

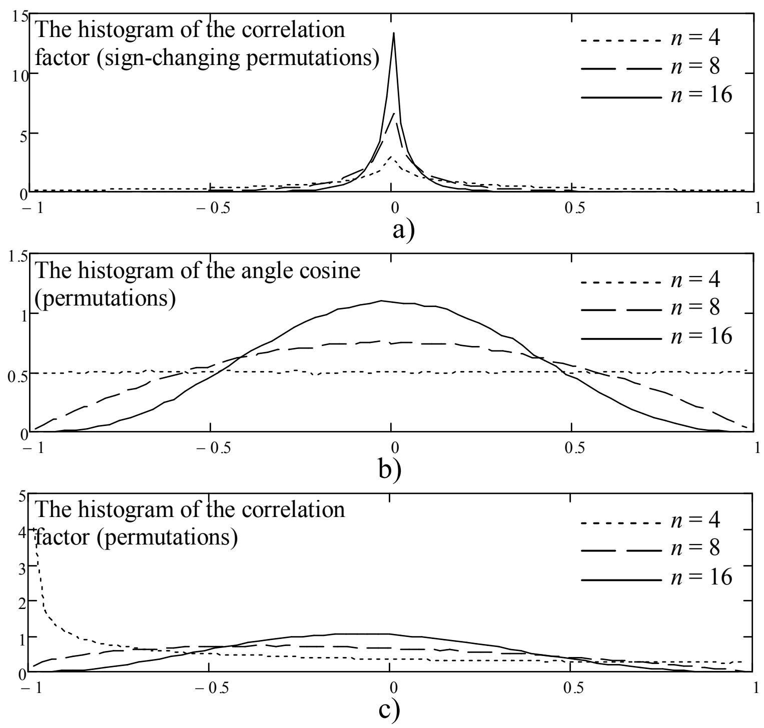

FIG. 2: The histograms of empirical distributions of the correlation factor between the images of different orthogonal sign-changing permutations (a), the angle cosine between the images of different permutations (b) and the correlation factor between the images of different permutations (c)

4. CONCLUSIONS

The present work is basically concerned with the way the maximal system of linear orthogonal transformations. This particular system as applicable to a specified nonzero discrete signal (vector) permits one to acquire a set of mutually orthogonal signals. This system of orthogonal signals can be utilized to ensure the code division of channels in the noise-signal phase-shift keying communication systems [6].

It is shown herein that the matrix representation of operators for the constructed transformation system can be derived as a set of basic γ -matrices of negative signature for the real matrix representation of the maximal Clifford algebra.

Based on this correspondence a maximal number of operators in the system of transformations with preset space dimension is ascertained. In order to get the operator matrices properly computed a series of explicitly notated formulas is provided.

We have demonstrated that technically the notation of the transformation system as a set of orthogonal sign-changing permutation is the most efficient one. We have studied the statistical properties of the constructed system of transformations and received experimental evidence in support of anticipated properties of the system.

We have highlighted the advantage of this particular system over the permutation procedures [5].

REFERENCES

1. Ilchenco, M.Ye., Kalinin, V.I., Narytnik, T.N., and Cherepenin, V.A., (2011), Wireless UWB Ecological Safety Communications at 70 Nanowatt Radiation Power, *CriMiCo'2011, Conf. Proceedings*, Sevastopol, 1:355-356 (in Russian).
2. Narayanan, R.M. and Chuang, J., (2007), Covert communications using heterodyne correlation random noise signals, *Electronics Letters*, **43**(22):1211-1212.
3. Kolumbán, G., Vizvari, B., Schwarz, W., and Abel, A., (1996), Differential chaos shift keying: A robust coding for chaos communications, *4-th Int. Workshop on Nonlinear Dynamics of Electronics Systems, (NDES'96)*, Seville, Spain, pp. 87–92.
4. Pervuninsky, S.M., Didfovsky, R.M., Metelap, V.V., and Tobilevich, Yu.Ye., (2006), Mathematical simulation of communication systems with correlation-temporal modulation, *Prikladnaya methmatika*, 83:112-123 (in Russian).
5. Lau, F.C.M., Cheong, K.Y., and Tse Chi, K., (2003), Permutation-Based DCSK and Multiple-Access DCSK Systems, *IEEE Trans. Circuits Syst. I*, **50**(6):733–742.
6. Didkovsky, R.M., (2010), Improvement of noise immunity of the noise-signal phase-shift keying communication system, *Nauchnye zapiski UNIIS*, 2:23-31 (in Russian).
7. Lang, S., (2002), *Algebra*, Springer-Verlag, – 914 p.
8. Postnikov, M.M., (1989), *Lectures on algebraic topology. Theories of homotopies of cellular spaces*, Nauka, Moscow: 336 p. (in Russian).
9. Postnikov, M.M., (1982), *Lectures on geometry. Groups and Lie algebras*, Nauka, Moscow: 447 p. (in Russian).
10. Toppan, F., (2004), *Division Algebra, Generalized Supersymmetries, and Octonionic M-theory*, CBPF - Centro Brasileiro de Pesquisas Físicas, Notas de física, 031/04. – 17 p.