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TELECOMMUNICATIONS

POTENTIAL PERFORMANCE OF THE COMMUNICATION SYSTEMS USING AUTOCORRELATION RECEPTION OF SHIFT-KEYED NOISE SIGNALS

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Mathematical models of the signals peculiar to communications using the autocorrelation reception of the shift-keyed noise signals are suggested in the present paper. System performance is investigated and the particularities of the bit error probability dependence upon the normalized signal-to-noise ratio and the bandwidth-delay product are revealed. The definition of the optimal value of the signal-to-noise ratio upon power is substantiated and the relevant values are calculated. The performed comparative analysis proved that the communication using the differential noise shift-keying, within which total time delay diversity is applied between the reference and the data signals, possesses the best system performance among the communications considered in this paper.

KEY WORDS: *UWB communications, noise carrier, transmitted reference, autocorrelation receiving, system performance, correlation-delay noise shift-keying, antipodal noise signals, differential noise shift-keying, bit error probability, bandwidth-delay product*

1. INTRODUCTION

The middle of the last century was marked with an active search for new principles and carriers for telecommunication. In this respect it needs to be recalled the pioneer work of academician A.A. Kharkevich [1], in which the concept of using the noise (stochastic) signal as the means of data carrier has been originally suggested. His concept was supported by a number of authors [2-4] and developed towards the direction of using noise signals for wireless transmission of discrete information.

At the beginning of 1970s a sort of summarizing of the studies accomplished by that period in the sphere of broadband communications (including those applying the noise data carrier) used to be performed in paper [5]. An attempt to analyze the issue of potential system performance of that kind of communications was also made in the above paper. However, the assumption about the Gaussian distribution of the random value in the pre-detector point of the receiver resulted in a systematical error in calculation of the information bit error probability during the reception (towards increasing of the value).

Next wave of the researcher interest to the communications using the noise signals re-appeared only in the late 20th – early 21st centuries [6-9]. It is related to the new opportunities opened up due to improvement of the component base of radio equipment and wide implementation of digital techniques for forming and processing of signals.

Research activities performed during recently in the sphere of communications using the noise carrier are stipulated primarily by the ever growing demand for protection of the data transmitted in the communication channel [10-11] and environmental safety of the radiation [12]. It is also not less important to explore new bands of ultra-high frequencies because noise signals in the sub-terahertz band are considered as a quite promising kind of the carrier [13].

Development of an adequate theory for the potential system performance of one or another communication is the necessary condition for its successful realization and implementation. Availability of a theory allows clarification at the system design stage the perspectives of the studies in this sphere and the optimality level of the suggested solutions, and at the operation stage – selection of optimal parameters of the system depending upon the conditions of the interference environment.

It should be noted that calculation of the information bit error probability during the data transmission/reception for broadband communications using the stochastic carrier signal is a rather complicated mathematical and computing problem that has not been solved before with a sufficient degree of stringency.

Recovering of the reference broadband noise signal and creation of a copy thereof with the help of a local generator in a remote broadband communication system receiver does not seem possible [14]. There are two possible directions of solving the above problem:

1. Application of the noise signal shift-keying techniques allowing power or generalized power reception [15-16,10], for example, amplitude or frequency shift-keying.

2. Application of the transmitted reference – TR technique – when the reference and the data noise signals are transmitted simultaneously into the wireless telecommunication link and can be separated upon the spectrum, time delay or orthogonal polarizations of the radiated signal [11,14,17].

Communications using reference signal transmission possess the best parametric security characteristics among the communications using the noise carrier. It is especially important for providing data transmission confidentiality. Therefore, just the above type of communications will be the subject for investigation.

Time delay diversity of the reference and the data signals while shift-keying of noise signals in the transmitter [9,11] is studied in our paper. In this case introduction of digital data into the noise signal occurs by means of formation of the secondary maximum of the signal autocorrelation function and controlling the signal parameters (time positioning or polarity).

Optimal noise signals processing in the transmitter includes calculation of certain values of the correlation function for the reference noise signal and the noise carrier shift-keyed in accordance with the transmitted data flow.

Correlation processing of the signals in TR system receivers is performed synchronously to the incoming bit flow that is essentially decreasing the requirements set to the synchronization accuracy and allows using simple schemes for development of wireless communications. However, TR radio communication systems are inferior to the spread spectrum coherent systems in terms of the system performance because the transmitted power is distributed to provide for simultaneous transmission of both the data and the reference signals [11,14,17].

2. COMMUNICATION USING THE CORRELATION-DELAY NOISE SHIFT-KEYING

Communication links using the correlation-delay noise shift-keying (CDNSK) are among the most thoroughly studied from both theoretical and experimental points of view. At that, they are studied in various frequency bands [2-3,8-9,12-13].

Let $x(t)$ be the signal at the output of the noise signal generator (NSG). We assume that $x(t)$ is the realization of the continuous stochastic process having a homogeneous amplitude spectrum within the frequency bandwidth $\left[f_1; f_2\right]$, zero mathematical expectation and the dispersion (power) equal to D_x.

We designate as $k = 1, 2, 3, ...$ the number of the current bit interval, the duration of each of them is equal to T, and α_k as the information bit (0 or 1) transmitted upon a given interval. Then the mathematical model of the signal at the output of the transmitter of the communication using CDNSK can be put down as follows

$$y(t) = x(t) + \left(1 - \alpha_k\right) x\left(t - \tau_0\right) + \alpha_k x\left(t - \tau_1\right), \; t \in \left[\left(k-1\right) \cdot T, k \cdot T\right]. \quad (1)$$

Represented in Fig. 1 is a simplified block diagram showing the units providing for formation and realization of the signal of the kind (1).

Changeover of the switch 7 under the input binary sequence law α_k occurring at the beginning of each bit interval provides for formation of the data signal that is in itself a copy of the reference signal $x(t)$, delayed by the line 5 for the time τ_0 (at transmission of zero) or by the line 6 for the time τ_1 (at transmission of one). Propagating through the power adder the reference signal is continuously radiated by the transmitter (the first summand in formula (1)).

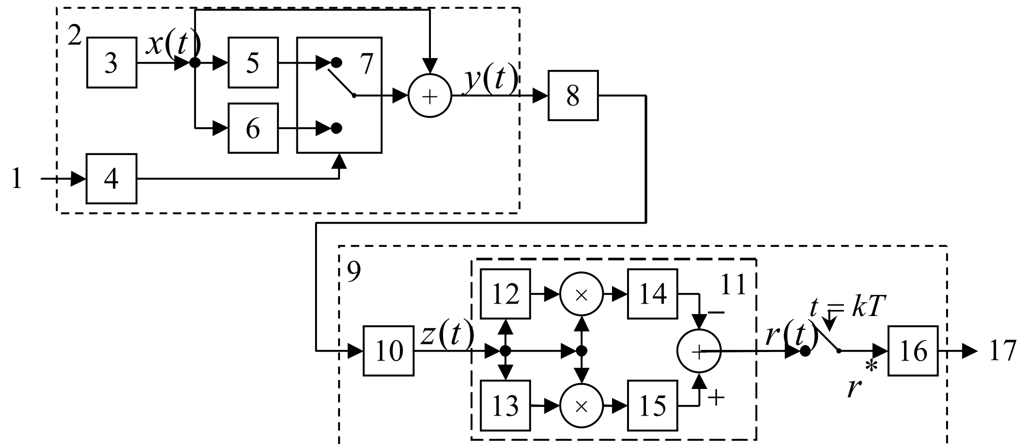

FIG. 1: Block diagram of the communication using CDNSK: 1 – transmitted bit; 2 – transmitter; 3 – noise signal generator (NSG); 4 – encoder; 5, 12 – delay line for τ_0; 6, 13 – delay line for τ_1; 7 – switch; 8 – channel; 9 – receiver; 10 – bandpass filter; 11 – demodulator; 14, 15 – integrating unit; 16 – detector; 17 – received bit

It must be noted that the time delays τ_0 and τ_1 along with the difference between them $|\tau_1 - \tau_0|$ have to be substantially larger than the signal correlation interval $x(t)$.

The signal $y(t)$ (as the sum of the reference and the data signals) acquires a secondary maximum of the autocorrelation function $R(\tau)$ (see Fig. 2). Just the position of the above maximum is the informational parameter of the signal. Hence it follow that the optimal technique for reception of the signal using CDNSK is in comparison of the values of the autocorrelation function calculated in two points τ_0 and τ_1.

This technique is realized with the help of the demodulator 11 (see Fig. 1) consisting of two autocorrelation filters (the time delay line, the mixer, and the integrating unit) tuned to the time delays τ_0 and τ_1, and of the subtracting unit.

Therefore, the signal of the following representation is observed at the demodulator output

$$r(t) = \int_{t-T}^{t} z(\tau)\cdot\left(z\left(\tau-\tau_1\right)-z\left(\tau-\tau_0\right)\right) d\tau \,,$$

where $z(t)$ is the additive mix of the payload signal $y(t)$ and noise $n(t)$ propagated through the input filter 10 with the throughput bandwidth $[f_1; f_2]$.

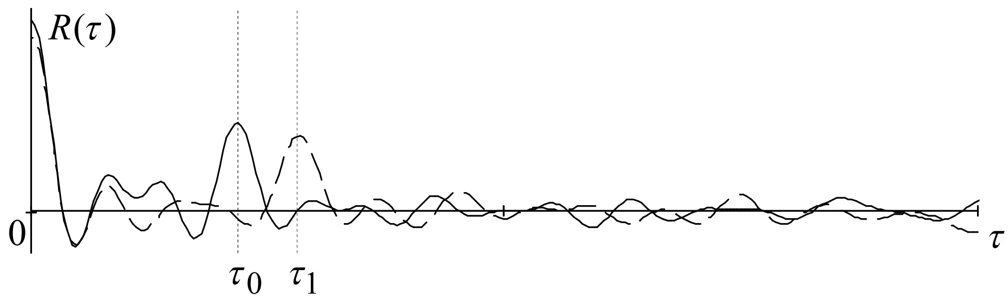

FIG. 2: Typical shape of the autocorrelation function $R(\tau)$ of the signal of the communication using CDNSK at transmission of the bit 0 (———) and 1 (– – – –)

In this case for the channel 8 we apply the classical model of the loss-free channel where there are operating additive interferences in the form of the Gaussian white noise with the one-side spectral density N_0. Then the signal $n(t)$ is the realization of the Gaussian random process with the frequency bandwidth $[f_1; f_2]$, zero mathematical expectation and the dispersion $D_n = N_0 \cdot F$, where $F = f_2 - f_1$ is the width of the signal frequency band.

At the end of the bit interval (at $t = kT$) the gating unit controlled by the synchronization system performs selection of the value of the function $r(t)$. The value of $r^* = r(kT)$ is fed to the input of the threshold detector 16. If $r^* > 0$ then it is accepted the hypothesis H_1 about receiving of one, otherwise – the hypothesis H_0 about receiving of zero.

We designate as $r^*(0)$ the value r^* under the condition that $\alpha_k = 0$, and $r^*(1)$ – under the condition that $\alpha_k = 1$. Then

$$\begin{aligned} r^*(0) = \int_{(k-1)\cdot T}^{k\cdot T} & \left(x(t) + x(t-\tau_0) + n(t)\right) \times \\ & \times \left(x(t-\tau_1) + x(t-\tau_0-\tau_1) + n(t-\tau_1) - \left(x(t-\tau_0) + x(t-2\tau_0) + n(t-\tau_0)\right)\right) dt, \end{aligned} \tag{2}$$

$$\begin{aligned} r^*(1) = \int_{(k-1)\cdot T}^{k\cdot T} & \left(x(t) + x(t-\tau_1) + n(t)\right) \times \\ & \times \left(x(t-\tau_1) + x(t-2\tau_1) + n(t-\tau_1) - \left(x(t-\tau_0) + x(t-\tau_1-\tau_0) + n(t-\tau_0)\right)\right) dt. \end{aligned}$$

Under the condition of equiprobable occurrence of the bits 0 and 1 within the input bit flow and the simple loss function, the data bit transmission/reception error probability P_b is calculated from the formula

$$P_b = \frac{1}{2} \cdot \left(P\left(r^*(0) > 0\right) + P(r^*(1) < 0) \right). \tag{3}$$

Under the above conditions superimposed upon the signals $x(t)$ and $n(t)$ the probability distributions of the random values $r^*(0)$ and $r^*(1)$ are symmetrical with respect to zero. Therefore, in order to calculate P_b it would be sufficient to study the distribution $r^*(1)$, and the formula (3) could be re-written as follows

$$P_b = P\left(r^*(1) < 0\right). \tag{4}$$

Considering that the demodulator input signal $z(t) = y(t) + n(t)$ possesses a limited spectrum then using the Kotelnikov theorem we proceed to the discrete time scale with the sampling period of $\Delta t = 1/\left(2F\right)$. After that the signals contained in (2) will be represented by real vectors, the dimensions of which upon one bit interval will be equal to $N = 2B$, where $B = FT$ is the bandwidth-delay product.

We designate x_j, $j = 1, 2, ..., N$ as the signal $x(t)$ counts upon the k-th bit interval, n_j as the counts of the signal $n(t)$. We shall designate with different number of primes the counts of the signals delayed for a different time. Then the discrete analog of the formula (2) will be

$$r^{*\Delta}(1) = \sum_{j=1}^{N} \left(x_j + x_j' + n_j \right) \left(\left(x_j' + x_j'' + n_j' \right) - \left(x_j''' + x_j'''' + n_j'' \right) \right). \tag{5}$$

To simplify the process of subsequent computation the scale multiplier Δt is omitted in this expression.

At $N = 1$ the expression (5) can be briefly put down as follows

$$r^{*\Delta}(1) = (x' + u)(x' + v),$$

where x', u and v are the Gaussian random values with zero mathematical expectation and the dispersions of D_x, $D_u = D_x + D_n$ and $D_v = 3D_x + 2D_n$ correspondingly.

Then the function $p_{r(1)}(v)$ of probability distribution density of the value $r^{*\Delta}(1)$ at $N = 1$ will have the following representation

$$p_{r(1)}(\nu)\Big|_{N=1} = \frac{1}{\pi\sqrt{D}} \cdot \exp\left(\frac{D_x \nu}{D}\right) \cdot K_0\left(\frac{\sqrt{D_x^2 + D}}{D} \cdot |\,\nu\,|\right), \qquad (6)$$

where $D = D_x D_u + D_x D_v + D_u D_v$, K_0 is the modified Bessel function of the second kind of the zero order. Proving of this statement is provided in the Annex.

To find the function $p_{r(1)}(\nu)$ at an arbitrary N we use the set of characteristic functions [18].

The characteristic function of a random value $r^{*\Delta}(1)$ at $N = 1$ is determined as the inverse Fourier transform for the function $p_{r(1)}(\nu)\Big|_{N=1}$

$$\varphi(t)\Big|_{N=1} = \int_{-\infty}^{\infty} e^{it\nu} \cdot p_{r(1)}(\nu)\Big|_{N=1} \, d\nu. \qquad (7)$$

It is important to mention that the characteristic function of the sum of independent random values is equal to the product of the respective characteristic functions, thus

$$\varphi(t)\Big|_{N=k} = \varphi^k(t)\Big|_{N=1}. \qquad (8)$$

The unicity theorem and the inverse transform theorem are true for the characteristic functions therefore, the density function is determined unambiguously

$$p_{r(1)}(\nu)\Big|_{N=k} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-it\nu} \, \varphi(t)\Big|_{N=k} \, dt. \qquad (9)$$

Sequential performance of calculations upon the formulas (7)-(9) solves the problem of finding of the distribution density function $r^{*\Delta}(1)$ at an arbitrary N.

Then the equality (4) can be re-written in the form suitable for direct calculations of the error probability

$$P_b = \int_{-\infty}^{0} p_{r(1)}(\nu) \, d\nu \, . \qquad (10)$$

Geometrical interpretation of the formula (10) is represented in Fig. 3. The probability P_b is equal to the area of the curvilinear triangle *ABO*. We know that the point *A* is marked in the Figure conditionally. In reality, it is infinitely remote to the left.

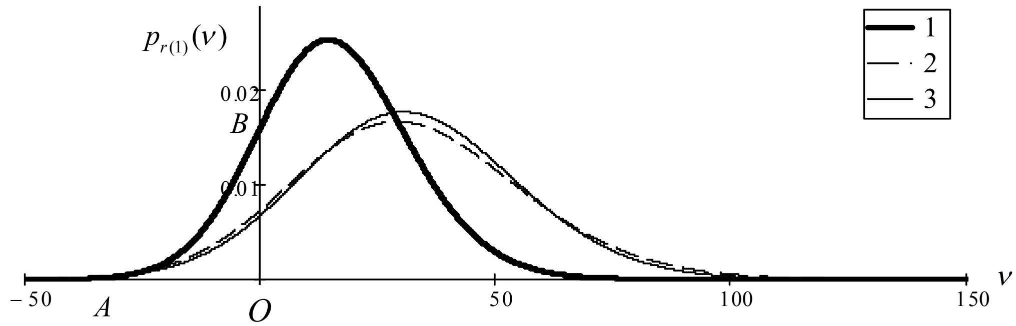

FIG. 3: Random value distribution density function at the input of the detector of the communication using CDNSK at various values of the signal parameters (1 – $N=32$, $D_x=1/2$, $D_n=1$; 2 – $N=32$, $D_x=1$, $D_n=1$; 3 – $N=64$, $D_x=1/2$, $D_n=1$)

Dependence of the bit error probability P_b upon the normalized value of the signal-to-noise ratio [19] is the classical indicator of a digital communication system antijamming performance

$$h^2=\frac{E_b}{N_0}=\frac{D_y\cdot T}{D_n/F}=\frac{D_y}{D_n}\cdot FT=\rho^2\cdot B\,, \tag{11}$$

where E_b is the information bit transmission power, $\rho^2=\dfrac{D_y}{D_n}$ is the signal-to-noise ratio upon power, and B is the bandwidth-delay product.

However, the information provided in Fig. 3 allows stating that that for the system under investigation increasing of the signal power by, for example, two times and double increase of the signal duration (that provides for an equivalent increment of the bit transfer power) provide for a different result both from the point of view of the shape of the function $p_{r(1)}(\nu)$ and of the bit error probability P_b.

Analysis of the expressions (6)-(10) based on which P_b is calculated, demonstrates that in this case the bit error probability is a function of two independent variables ρ^2 and B. Performing simple transforms the dependence of $P_b=P_b\left(\rho^2,B\right)$ can be put down in a more customary representation $P_b=P_b\left(h^2,B\right)$. The results of the relevant calculations are illustrated in Fig. 4.

As it is evident from Fig. 4 the surface of $P_b=P_b\left(h^2,B\right)$ possesses a rather complex shape; in doing so if the dependences of P_b upon h^2 at the fixed B possess a traditional waterfall-like shape (Fig. 5(a)) then graphs of the dependences of P_b upon

B at the fixed h^2 have a complex shape with the only minimum and the horizontal asymptote (Fig. 5(b)).

The value of the bandwidth-delay product, at which the minimum of the bit error probability is attained for the set h^2, is called the optimal the bandwidth-delay product and designated as B_{opt}.

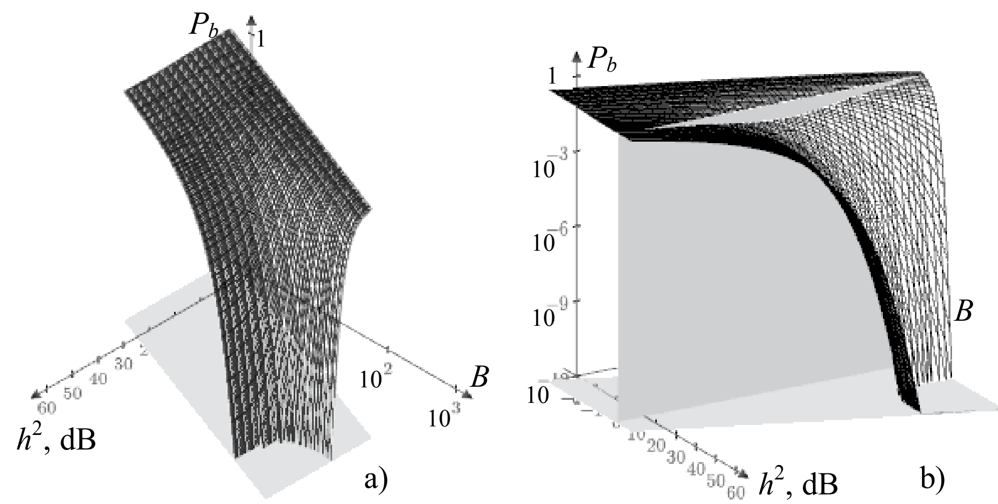

FIG. 4: Graph of the dependence of $P_b = P_b\left(h^2, B\right)$ for the communication using CDNSK (a) and the surface cross-section at $h^2 = 15.05$ dB (b)

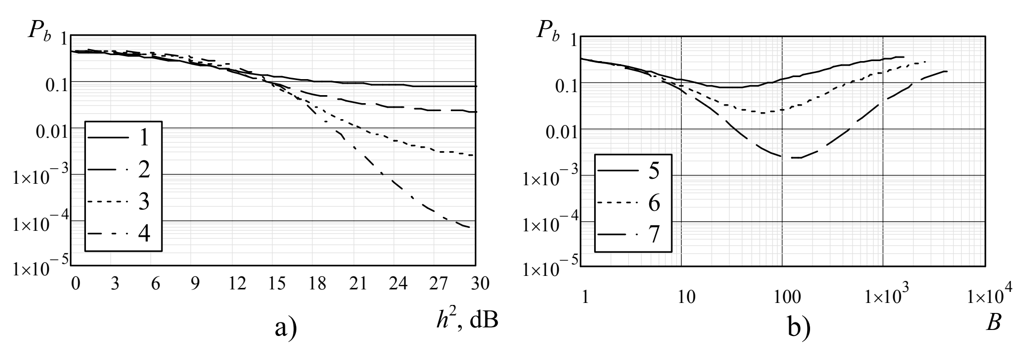

FIG. 5: Dependence of $P_b = P_b\left(h^2\right)$ at the fixed bandwidth-delay product B (1 - $B = 8$; 2 - $B = 16$; 3 - $B = 32$; 4 - $B = 64$) (a) and dependence of $P_b = P_b(B)$ at the fixed h^2 (5 - $h^2 = 15.051$ dB, 6 - $h^2 = 18.062$ dB, 7 - $h^2 = 21.072$ dB) (b) for the communication system using CDNSK

The calculations show that between the value of h^2 and the optimal bandwidth-delay product B_{opt} there exists a simple linear relationship of the following kind

$$B_{opt} \approx k \cdot h^2 . \tag{12}$$

From (11) and (12) there follows the meaning of the optimal value of the signal-to-noise ratio upon power $\rho^2_{opt} = 1/k$. We have to note that the value of k in (12) and, thus, of ρ^2_{opt} as well, depends solely upon the noise shift-keying technique. In particular, for the communication using CDNSK we have: $k = 1.06$, $\rho^2_{opt} = 0.943$.

3. COMMUNICATION SYSTEM USING THE ANTIPODAL NOISE SIGNALS

Now let us consider the communication using the antipodal noise signals (ANS) suggested in [4]. Block diagram of the communication is provided in Fig. 6.

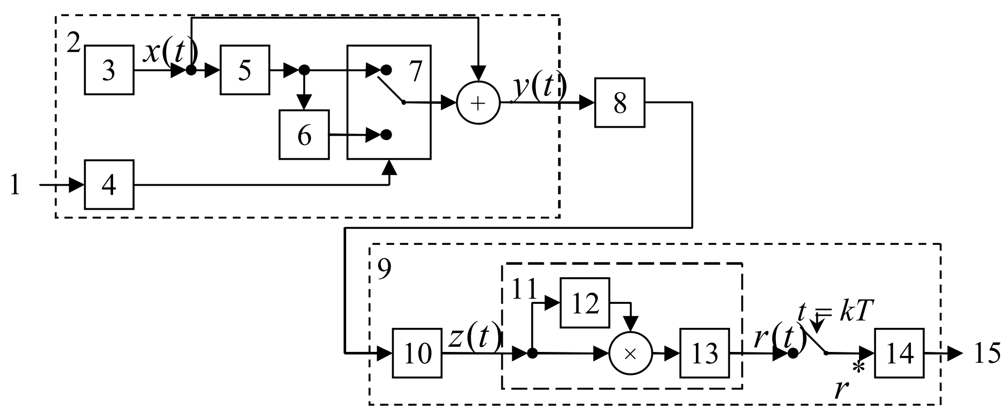

FIG. 6: Block diagram of the communication system using ANS: 1 – transmitted bit; 2 – transmitter; 3 – noise signal generator (NSG); 4 – encoder; 5, 12 – delay line for τ_0 ; 6 – phase shifter; 7 – switch; 8 – channel; 9 – receiver; 10 – bandpass filter; 11 – demodulator; 13 – integrating unit; 14 – detector; 15 – received bit

The receiver of the communication system using ANS contains one delay line only. In this case the information is encoded not by means of shifting the position of the secondary maximum of the autocorrelation function, but by shifting the polarity of the said maximum (see Fig. 7).

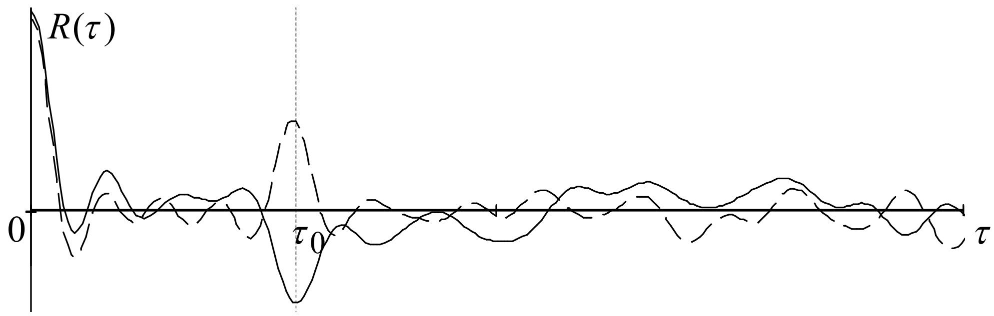

FIG. 7: Typical shape of the autocorrelation function $R(\tau)$ of the signal of the communication system using ANS at transmission of the bit 0 (———) and 1 (– – – –)

Structurally, the transmitter of the communication system is different from the one considered above because the second delay line is replaced by the broadband phase shifter 6 with phase shifting by 180°. We admit that the phase shifter has to provide for a fixed phase shifting within the operation frequency bandwidth $\left[f_1;f_2\right]$.

At the output of the transmitter 2 we obtain the signal, the mathematical model of which can be put down in the following way

$$y(t)=x(t)+\left(2\alpha_k-1\right)x\left(t-\tau_0\right),\ t\in\left[\left(k-1\right)\cdot T,k\cdot T\right].$$

Considering that in this case the time delay between the reference and the data signals is constant (independent upon the current bit α_k), and changing of the autocorrelation function sign in the point of measurement is set in the structure of the signal itself, the system receiver 9 is simplified essentially. It contains one autocorrelation filter tuned for the time delay of τ_0.

At the output of the demodulator 11 there will be the signal

$$r(t)=\int_{t-T}^{t} z(\tau)\cdot z\left(\tau-\tau_0\right)d\tau .$$

Then the following value will arrive at the detector input in the end of the bit interval

$$r^*=\int_{(k-1)\cdot T}^{k\cdot T}\left(x(t)+\left(2\alpha_k-1\right)x\left(t-\tau_0\right)+n(t)\right)\times$$

$$\times\left(x\left(t-\tau_0\right)+\left(2\alpha_{k-1}-1\right)x\left(t-2\tau_0\right)+n\left(t-\tau_0\right)\right)dt.$$

A $\alpha_k = 1$ a discrete analog of this expression will be as follows

$$r^{*\Delta}(1) = \sum_{j=1}^{N} \left(x_j + x'_j + n_j\right)\left(x'_j \pm x''_j + n'_j\right),$$

and the probability distribution density function for the value $r^{*\Delta}(1)$ at $N = 1$ will be put down as follows

$$p_{r(1)}(\nu)\Big|_{N=1} = \frac{1}{\pi\sqrt{3D_x^2 + 4D_x D_n + D_n^2}} \cdot \exp\left(\frac{D_x}{3D_x^2 + 4D_x D_n + D_n^2}\nu\right) \times \\ \times K_0\left(\frac{2D_x + D_n}{3D_x^2 + 4D_x D_n + D_n^2}|\nu|\right). \tag{13}$$

Subsequent calculations of the bit error probability P_b are performed similar to the communication using CDNSK upon the formulas (7)-(10) with substitution of (13). The obtained results are illustrated in Fig. 8.

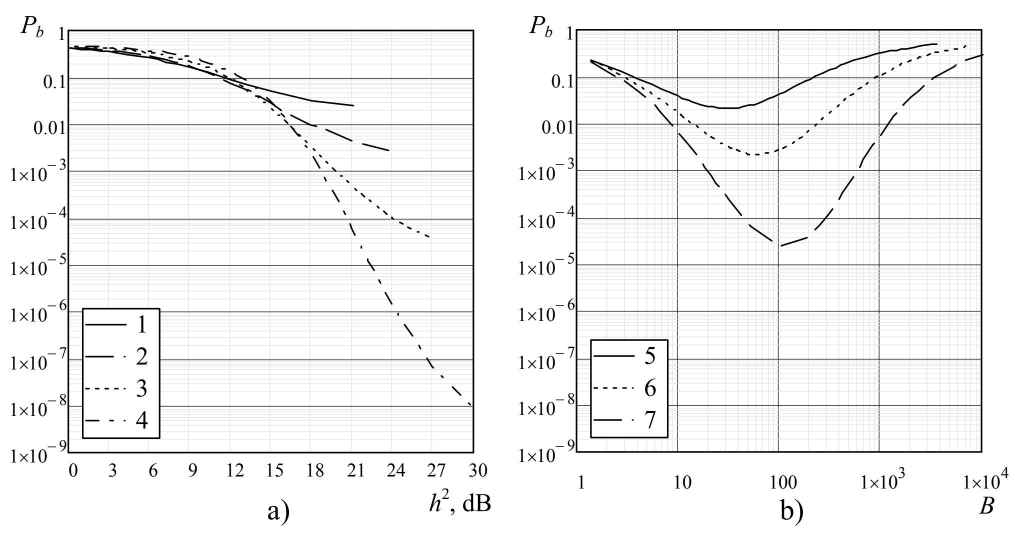

FIG. 8: Dependence of $P_b = P_b\left(h^2\right)$ at the fixed bandwidth-delay product B (1 - $B = 8$; 2 - $B = 16$; 3 - $B = 32$; 4 - $B = 64$) (a) and dependence of $P_b = P_b(B)$ at the fixed h^2 (5 - $h^2 = 15.051$ dB, 6 - $h^2 = 18.062$ dB, 7 - $h^2 = 21.072$ dB) (b) for the communication system using ANS

As it is evident from the Figure particularities of the dependence of P_b upon h^2 and B specified above for CDNSK remain for ANS as well.

However, the value of the coefficient k in the formula (12) is, in this case, equal to 1.118. Therefore, for the communication using ANS $\rho_{opt}^2 = 0.894$.

It should be noted that under the equal conditions the bit error probability in the communication system using ANS is substantially lower than in the communication using CDNSK (Figs. 5 and 8). Thus, using of the antipodal signals, like in the classical case, allows obtaining a gain in system performance.

A common particularity of the dependences of $P_b = P_b\left(h^2\right)$ (at the fixed bandwidth-delay product) obtained for the communications using CDNSK and ANS is availability of the non-zero horizontal asymptotes on the graphs of the above dependences. This particularity is stipulated by the fact that both the reference and the data signals are transmitted in the channel of both communications simultaneously within one and the same frequency band (although with a time shift). These signals create interference for each other (intra-system noise). As the result, the bit error probability of the receiver appears to be larger than zero even if there is absolutely no interference in the communication channel (i.e., at $D_x = const$, $D_n \to 0$, $h^2 \to +\infty$) or at whatever high power of the transmitter signal ($D_n = const$, $D_x \to +\infty$, $h^2 \to +\infty$), if the bandwidth-delay product value is fixed ($B = const$).

The bit error probability in the communications using CDNSK and ANS can be tended to zero solely by means of increasing the bandwidth-delay product. However, the data transfer rate (if the signal frequency bandwidth is fixed) is, at that, also tending to zero.

4. COMMUNICATION SYSTEM USING DIFFERENTIAL NOISE SHIFT-KEYING

The problem of intra-system noise is solved in the communication using the differential noise shift-keying (DNSK) suggested in the paper [20]. Block diagram of the communication is provided in Fig. 9.

The communication system using DNSK applies the technique for time-sharing of the reference and the data signals. A power combiner is absent from the transmitter. The three-positional switch 7 is applied in its place. During the first half of the bit interval the switch short-circuits the transmitter output directly upon the NSG. In such a manner the reference signal has shaped.

In the middle of the bit interval there occurs cutover of the switch into one of the two possible positions (with or without phase shifting) depending upon the signal from the encoder 4. At that, the delay line 5 provides for delaying of the signal $x(t)$ by a half of the bit interval $T/2$. As a result the data signal has shaped. Therefore, the mathematical model of the transmitter output signal is as follows

$$y(t)=\begin{cases} x(t), & t\in\left[(k-1)\cdot T,(k-1/2)\cdot T\right], \\ (2\alpha_k-1)x\left(t-T/2\right), & t\in\left[(k-1/2)\cdot T,k\cdot T\right]. \end{cases}$$

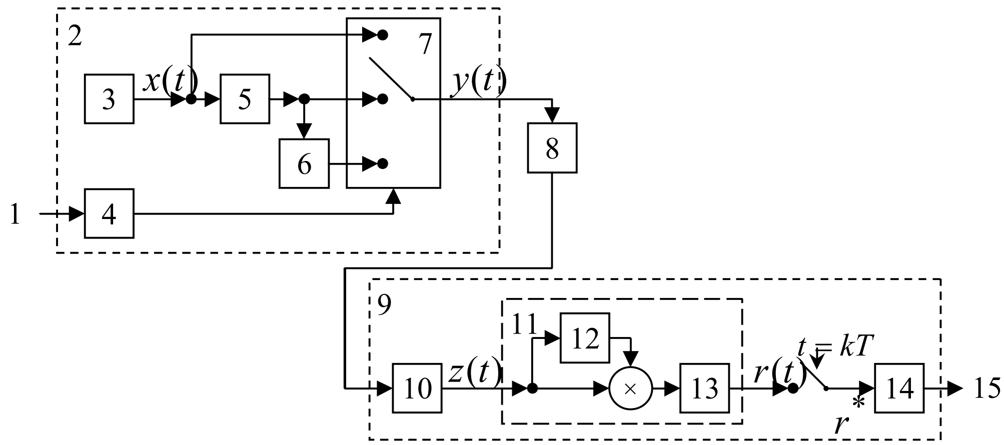

FIG. 9: Block diagram of the communication system using DNSK: 1 – transmitted bit; 2 – transmitter; 3 – NSG; 4 – encoder; 5, 12 – delay line for $T/2$; 6 – phase shifter; 7 – switch; 8 – channel; 9 – receiver; 10 – bandpass filter; 11 – demodulator; 13 – integrating unit; 14 – detector; 15 – received bit

The receiver of the communication system using DNSK is not different in its structure from the receiver of ANS. The difference is that the integrating unit 13 possesses the integration period of $T/2$ instead of T and the delay line 12 delays the signal for the time $T/2$. At the demodulator output we have the following signal

$$r(t)=\int_{t-T/2}^{t} z(\tau)\cdot z\left(\tau-T/2\right)d\tau\ ,$$

and incoming signal at the detector input is

$$r^{*}=\int_{(k-1/2)\cdot T}^{k\cdot T}\left((2\alpha_k-1)x\left(t-T/2\right)+n(t)\right)\cdot\left(x\left(t-T/2\right)+n\left(t-T/2\right)\right)dt.$$

The discrete analog of this expression at $\alpha_k=1$ will be put down as follows

$$r^{*\Delta}(1)=\sum_{j=1}^{N}\left(x'_j+n_j\right)\left(x'_j+n'_j\right).$$

It should be mentioned that in this case N is the number of sampling counts along the length of the reference signal (not upon the bit interval). Therefore, in this case the bandwidth-delay product will be equivalent $B = N$.

At $N = 1$ the probability distribution density function of the value $r^{*\Delta}(1)$ will have the following representation

$$p_{r(1)}(\nu)\Big|_{N=1} = \frac{1}{\pi\sqrt{D_n\left(2D_x + D_n\right)}} \cdot \exp\left(\frac{D_x}{D_n\left(2D_x + D_n\right)}\nu\right) \times \\ \times K_0\left(\frac{\sqrt{D_x^2 + 2D_x D_n + D_n^2}}{D_n\left(2D_x + D_n\right)}|\nu|\right). \tag{14}$$

Substituting (14) into (7)-(10) we obtain the dependence of the bit error probability P_b upon the signal-to-noise ratio h^2 and the bandwidth-delay product B for the communication system using DNSK (see Fig. 10).

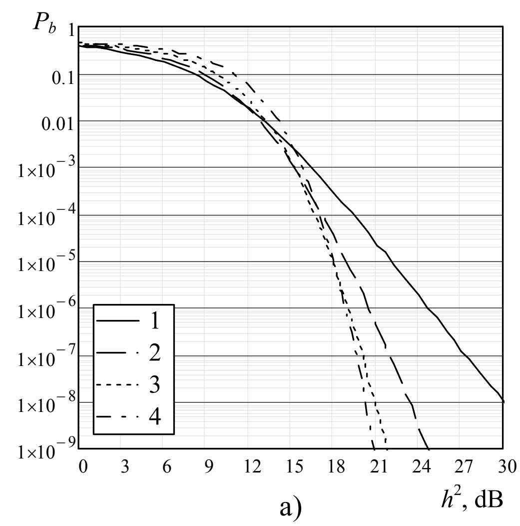

FIG. 10: Dependence of $P_b = P_b\left(h^2\right)$ at the fixed bandwidth-delay product B (1 - $B = 8$; 2 - $B = 16$; 3 - $B = 32$; 4 - $B = 64$) (a) and dependence of $P_b = P_b(B)$ at the fixed h^2 (5 - $h^2 = 15.051$ dB, 6 - $h^2 = 18.062$ dB, 7 - $h^2 = 21.072$ dB) (b) for the communication system using DNSK

All the graphs in Fig. 10(a) possess a zero right-hand side horizontal asymptote. Therefore, at increasing of h^2 (due to the increase of ρ^2 at the fixed bandwidth-delay product B) the bit error probability in the DNSK communication tends to zero (unlike in the communications using CDNSK and ANS) that is attained due to elimination of the intra-system interference.

For DNSK the coefficient k from (12) is $k = 0.687$, then $\rho_{opt}^2 = 1.456$.

5. COMPARATIVE ANALYSIS OF THE SYSTEM PERFORMANCES

Let us compare obtained results. Figure 11 allows getting an idea about the correlation between the data bit error probability value P_b during transmission/reception in the communications studied above. Upon the level of potential system performance the best parameters among the communications using the autocorrelation reception of noise signals were demonstrated by the DNSK communication followed by communications using ANS and CDNSK.

It has to be noted that the gain provided by DNSK in terms of the bit error probability (at the fixed bandwidth-delay product) upon the curves $P_b = P_b\left(h^2\right)$ is increasing with the increase of the signal-to-noise ratio h^2. At $B = 128$ and $h^2 = 21.072$ dB the DNSK communication has the bit error probability value which is by 4 orders less than that of ANS, while CDNSK loses two more orders to ANS.

The simplest comparison of the potential noise immunity of systems can be made if it is assumed that optimal parameters of the signal are set for each of the systems.

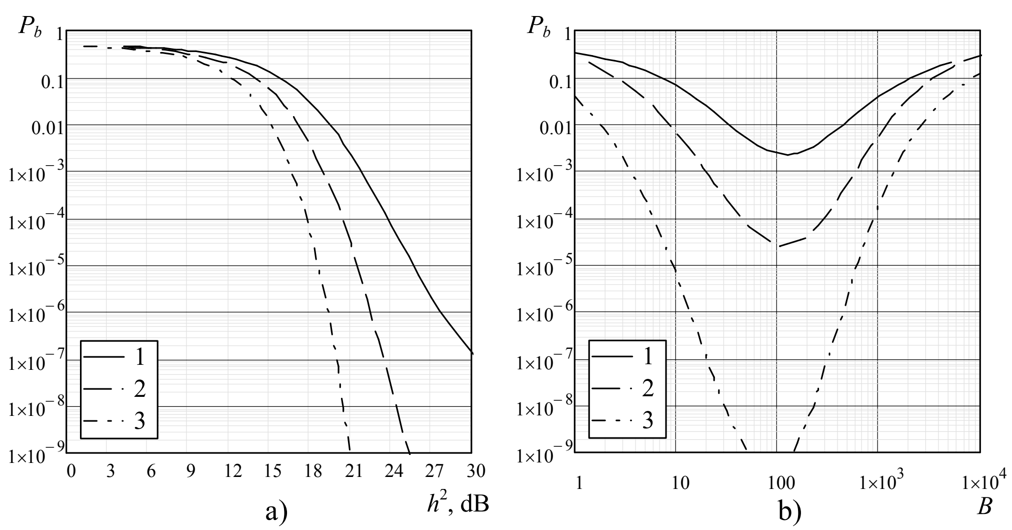

FIG. 11: Dependences of $P_b = P_b\left(h^2\right)$ at the fixed bandwidth-delay product $B = 128$ (a) and dependences of $P_b = P_b(B)$ at the fixed $h^2 = 21.072$ dB (b) for the communications using: 1 – CDNSK, 2 – ANS, 3 – DNSK

The procedure of setting optimal parameters for the communication using the noise carrier includes two stages:

1) setting of the transmitter power providing for the optimal value of the signal-to-noise ratio upon power;

2) selection of the bandwidth-delay product necessary for obtaining of the set value of h^2.

It should be noted that at optimal selection of the system parameters the function $P_b = P_b\left(h^2\right)$ is actually dependent upon variation of the bandwidth-delay product B at the fixed value of the signal-to-noise ratio upon power $\rho^2 = \rho^2_{opt}$. In this case the shape of the curve $P_b = P_b\left(h^2\right)$ has a classical representation that allows determining precise correlations between the efficiency values of various modes of shift-keying. Rated values of ρ^2_{opt}, loss of system performance for various communications using noise signals compared to the communication using the binary phase shift-keying (PSK) of the deterministic signal (coherent reception) as well as compared to the communication using DNSK are provided in the Table 1 below.

TABLE 1: Optimal values of the signal-to-noise ratio upon power and losses of the system performance at optimal parameters

Communication type	ρ^2_{opt}	With respect to PSK (coherent reception)		With respect to DNSK	
		times	dB	times	dB
CDNSK	0.943	32	15.051	4.734	6.752
ANS	0.894	16	12.041	2.367	3.742
DNSK	1.456	6.76	8.299	1	0

6. CONCLUSIONS

The mathematical models of signals and values typical for the communications using the autocorrelation reception of the shift-keyed noise signals are suggested in this paper.

The above models allowed performing a detailed study of system performances, revealing particularities of the dependences of the bit error probabilities upon the normalized values of the signal-to-noise ratio and the bandwidth-delay product. In particular, the definition of the optimal value of the signal-to-noise ratio upon power is substantiated and the relevant values are calculated. This definition is typical solely for the communications using stochastic (or chaotic) carrier signal.

The analysis shows that the DNSK communication provides for the best noise immunity among the communications considered in this paper. The advantage of the mentioned communication includes also simplification of the receiving unit as compared to the communication using CDNSK.

However, practical realization of the transmitters included into the communications using ANS and DNSK within the ultra-high frequency band (where the noise signals can be used the most efficiently) seems to be more complicated. This is explained by availability of a broadband phase sifter with the fixed frequency-independent shifting of the phase in the structure of the receiving unit. Development of this kind of unit for the above frequency band is quite a difficult problem to solve.

It should be mentioned that the communications using the noise carrier with transmission of reference signal and the autocorrelation reception provide for a high structural security level of the signal. Thus, the considered means of data transmission represent a special interest for development of the highly confidential communication systems.

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7. APPENDIX

7.1. Statement

Let as assume the

$$r = (x+n)(x+m), \tag{A1}$$

where x, n, m are the independent Gaussian random values with zero mathematical expectation and dispersions D_x, D_n and D_m correspondingly, then the probability distribution density function of the random value r has the following form

$$p_r(\nu) = \frac{1}{\pi\sqrt{D}} \cdot \exp\left(\frac{D_x \nu}{D}\right) \cdot K_0\left(\frac{\sqrt{D_x^2 + D}}{D} \cdot |\nu|\right),$$

where $D = D_n D_m + D_x D_m + D_x D_n$.

7.2 Proof

We express m from (A1) and find the partial derivative upon r:

$$m = \frac{r}{x+n} - x, \quad \frac{\partial m}{\partial r} = \frac{1}{x+n},$$

then in the general case we find

$$p_r(\nu) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_x(\xi) \cdot p_n(\theta) \cdot p_m\left(\frac{\nu}{\xi+\theta} - \xi\right) \cdot \frac{1}{|\xi+\theta|}\, d\xi d\theta. \tag{A2}$$

Considering that x, n and m possess the Gaussian distributions then $p_x(\xi) = \frac{1}{\sqrt{2\pi D_x}} \exp\left(-\frac{\xi^2}{2D_x}\right)$, $p_n(\theta) = \frac{1}{\sqrt{2\pi D_n}} \exp\left(-\frac{\theta^2}{2D_n}\right)$,

$$p_m\left(\frac{\nu}{\xi+\theta}-\xi\right)=\frac{1}{\sqrt{2\pi D_m}}\exp\left(-\frac{\left(\frac{\nu}{\xi+\theta}-\xi\right)^2}{2D_m}\right).$$

Substituting the above expressions into (A2) we obtain

$$p_r(\nu)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi D_x}}\exp\left(-\frac{\xi^2}{2D_x}\right)\cdot\frac{1}{\sqrt{2\pi D_n}}\exp\left(-\frac{\theta^2}{2D_n}\right)\times$$

$$\times\frac{1}{\sqrt{2\pi D_m}}\exp\left(-\frac{\left(\frac{\nu}{\xi+\theta}-\xi\right)^2}{2D_m}\right)\cdot\frac{1}{|\xi+\theta|}\,d\xi d\theta=$$

$$=\frac{1}{2\pi\sqrt{2\pi D_xD_nD_m}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(\underbrace{-\frac{\xi^2}{2D_x}-\frac{\theta^2}{2D_n}-\frac{\left(\frac{\nu}{\xi+\theta}-\xi\right)^2}{2D_m}}_{A}\right)\cdot\frac{1}{|\xi+\theta|}\,d\xi d\theta\,.$$

We transform the expression in the brackets

$$A=\frac{1}{2}\left(-\frac{\xi^2}{D_x}-\frac{\theta^2}{D_n}-\frac{\nu^2}{(\xi+\theta)^2D_m}+2\frac{\nu\xi}{(\xi+\theta)D_m}-\frac{\xi^2}{D_m}\right)=$$

$$=\frac{1}{2}\left(-\frac{\nu^2}{(\xi+\theta)^2D_m}-\frac{(\xi+\theta)^2}{D_n}+\frac{\xi^2}{D_n}+2\frac{\xi\theta}{D_n}-\frac{\xi^2}{D_x}-\frac{\xi^2}{D_m}+2\frac{\nu\xi}{(\xi+\theta)D_m}\right)=$$

$$=\frac{1}{2}\left(-\frac{\nu^2}{(\xi+\theta)^2D_m}-\frac{(\xi+\theta)^2}{D_n}+\frac{2\xi(\xi+\theta)}{D_n}-\xi^2\left(\frac{1}{D_x}+\frac{1}{D_n}+\frac{1}{D_m}\right)+2\frac{\nu\xi}{(\xi+\theta)D_m}\right).$$

We introduce the substitution $\begin{cases}\xi=\xi,\\ \zeta=\xi+\theta.\end{cases}$

We designate $B=\frac{1}{D_x}+\frac{1}{D_n}+\frac{1}{D_m}$, $C=\frac{1}{2\pi\sqrt{2\pi D_xD_nD_m}}$.

Then the integral acquires the following representation

$$=C\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(\frac{1}{2}\left(-\frac{\nu^2}{\zeta^2 D_m}-\frac{\zeta^2}{D_n}+\frac{2\xi\zeta}{D_n}-B\xi^2+2\frac{\nu\xi}{\zeta D_m}\right)\right)\cdot\frac{1}{|\zeta|}\,d\xi d\zeta=$$

$$=C\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(\frac{1}{2}\left(-\frac{\nu^2}{\zeta^2 D_m}-\frac{\zeta^2}{D_n}-B\xi^2+2\left(\frac{\zeta}{D_n}+\frac{\nu}{\zeta D_m}\right)\xi\right)\right)\cdot\frac{1}{|\zeta|}\,d\xi d\zeta=$$

We designate $E=\left(\dfrac{\zeta}{D_n}+\dfrac{\nu}{\zeta D_m}\right)$.

We pick up the complete square $-B\xi^2+2E\xi=-\left(\sqrt{B}\xi-\dfrac{E}{\sqrt{B}}\right)^2+\dfrac{E^2}{B}$. Then

$$=C\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(\frac{1}{2}\left(-\frac{\nu^2}{\zeta^2 D_m}-\frac{\zeta^2}{D_n}+\frac{E^2}{B}-\left(\sqrt{B}\xi-\frac{E}{\sqrt{B}}\right)^2\right)\right)\cdot\frac{1}{|\zeta|}\,d\xi d\zeta=$$

Let us consider the first three summands in the brackets

$$-\frac{\nu^2}{\zeta^2 D_m}-\frac{\zeta^2}{D_n}+\frac{E^2}{B}=-\frac{\nu^2}{\zeta^2 D_m}-\frac{\zeta^2}{D_n}+\frac{1}{B}\left(\frac{\nu^2}{\zeta^2 D_m^2}+\frac{\zeta^2}{D_n^2}+2\frac{\nu}{D_n D_m}\right)=$$

$$=-\left(\frac{1}{D_m}+\frac{1}{BD_m^2}\right)\frac{\nu^2}{\zeta^2}-\left(\frac{1}{D_n}+\frac{1}{BD_n^2}\right)\zeta^2+\frac{2\nu}{BD_nD_m}=$$

$$=-\frac{D_x+D_n}{D_nD_m+D_xD_m+D_xD_n}\cdot\frac{\nu^2}{\zeta^2}-\frac{D_x+D_m}{D_nD_m+D_xD_m+D_xD_n}\cdot\zeta^2+$$

$$+\frac{D_x}{D_nD_m+D_xD_m+D_xD_n}\nu.$$

We go back to the integral using the designation $D=D_nD_m+D_xD_m+D_xD_n$,

$$=C\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left(\frac{1}{2}\left(-\frac{D_x+D_n}{D}\cdot\frac{\nu^2}{\zeta^2}-\frac{D_x+D_m}{D}\cdot\zeta^2+\frac{D_x}{D}\nu-\left(\sqrt{B}\xi-\frac{E}{\sqrt{B}}\right)^2\right)\right)\cdot\frac{1}{|\zeta|}\,d\xi d\zeta=$$

The first two summands are independent upon ξ. The third summand is totally independent upon the integration variables. Let us proceed to the iterated integral (internal upon ξ and external upon ζ) by putting down the exponential of the sum as the product of exponentials. We obtain

$$= C \cdot \exp\left(\frac{D_x}{D}\nu\right)\int_{-\infty}^{+\infty} \exp\left(\frac{1}{2D}\left(-\left(D_x + D_n\right)\cdot\frac{\nu^2}{\zeta^2} - \left(D_x + D_m\right)\cdot\zeta^2\right)\right)\cdot\frac{1}{|\zeta|}d\zeta \times$$

$$\times\int_{-\infty}^{+\infty} \exp\left(\frac{1}{2}\left(-\left(\sqrt{B}\xi - \frac{E}{\sqrt{B}}\right)^2\right)\right)d\xi =$$

$$= \frac{C\sqrt{2\pi}}{\sqrt{B}}\cdot\exp\left(\frac{D_x}{D}\nu\right)\int_{-\infty}^{+\infty} \exp\left(\frac{1}{2D}\left(-\left(D_x + D_n\right)\cdot\frac{\nu^2}{\zeta^2} - \left(D_x + D_m\right)\cdot\zeta^2\right)\right)\cdot\frac{1}{|\zeta|}d\zeta \times$$

$$\times\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}}\exp\left(\frac{1}{2}\left(-\left(\sqrt{B}\xi - \frac{E}{\sqrt{B}}\right)^2\right)\right)d\left(\sqrt{B}\xi\right) =$$

The Gaussian distribution density function is under the internal integral sign. Thus, the said integral is equal to one. We have

$$= \frac{C\sqrt{2\pi}}{\sqrt{B}}\cdot\exp\left(\frac{D_x}{D}\nu\right)\int_{-\infty}^{+\infty} \exp\left(\frac{1}{2D}\left(-\left(D_x + D_n\right)\cdot\frac{\nu^2}{\zeta^2} - \left(D_x + D_m\right)\cdot\zeta^2\right)\right)\cdot\frac{1}{|\zeta|}d\zeta .$$

It is known that

$$\int_0^{\infty} \exp\left(-\frac{k}{y^2} - my^2\right)\cdot\frac{1}{y}dy = K_0\left(2\sqrt{km}\right),\ k, m > 0$$

or

$$\int_{-\infty}^{+\infty} \exp\left(-\frac{k}{y^2} - my^2\right)\cdot\frac{1}{|y|}dy = 2K_0\left(2\sqrt{km}\right),\ k, m > 0 .$$

Then

$$p_r(\nu) = \frac{C\sqrt{2\pi}}{\sqrt{B}}\cdot\exp\left(\frac{D_x}{D}\nu\right)\cdot 2K_0\left(\frac{\sqrt{\left(D_x + D_n\right)\left(D_x + D_m\right)}}{D}|\nu|\right).$$

Having performed elementary transforms we finally obtain

$$p_r(\nu) = \frac{1}{\pi\sqrt{D}}\cdot\exp\left(\frac{D_x\nu}{D}\right)\cdot K_0\left(\frac{\sqrt{D_x^2 + D}}{D}\cdot|\nu|\right),$$

where $D = D_n D_m + D_x D_m + D_x D_n$,
which proves the statement.